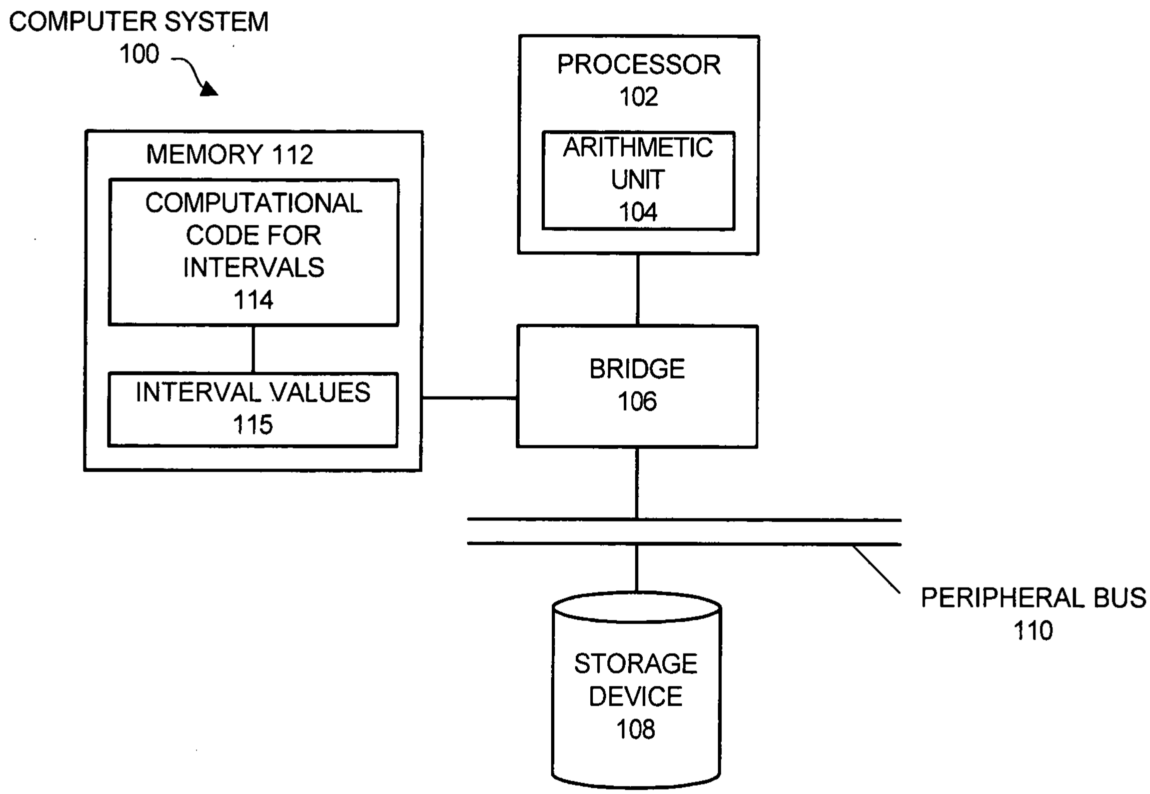
COMPUTER SYSTEM
100
MEMORY 112
COMPUTATIONAL CODE FOR INTERVALS
114
INTERVAL VALUES
115
PROCESSOR
102
ARITHMETIC UNIT
104
BRIDGE
106
STORAGE DEVICE
108
PERIPHERAL BUS
110

FIG. 1

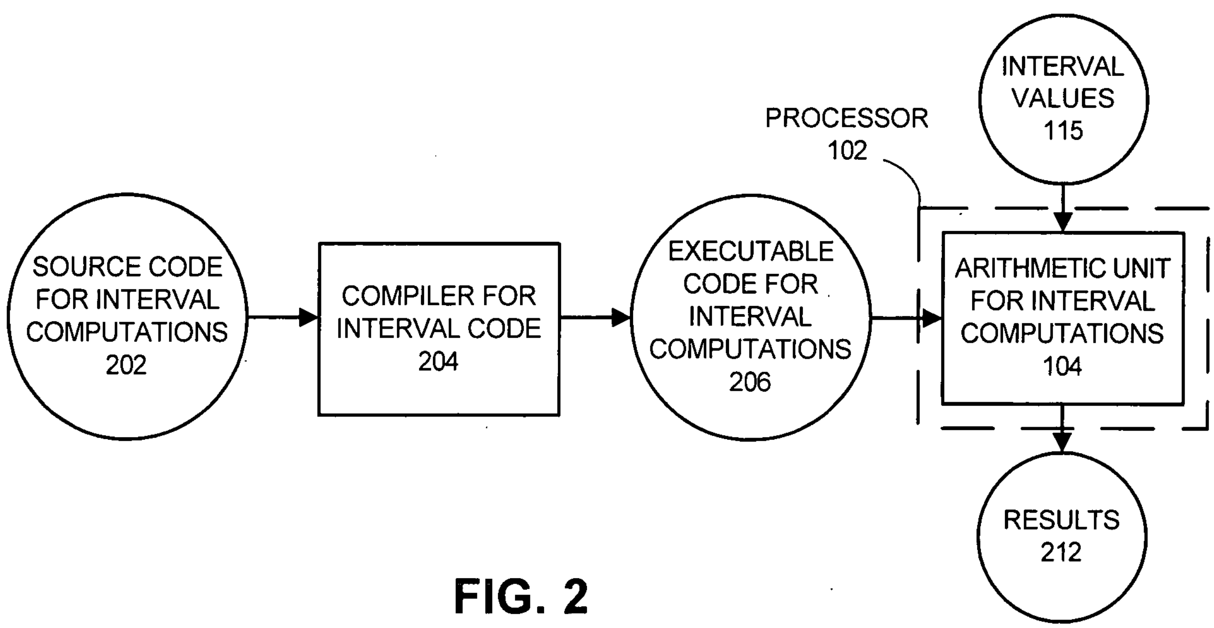
SOURCE CODE FOR INTERVAL COMPUTATIONS
202
COMPILER FOR INTERVAL CODE
204
EXECUTABLE CODE FOR INTERVAL COMPUTATIONS
206
PROCESSOR
102
INTERVAL VALUES
115
ARITHMETIC UNIT FOR INTERVAL COMPUTATIONS
104
RESULTS
212

FIG. 2

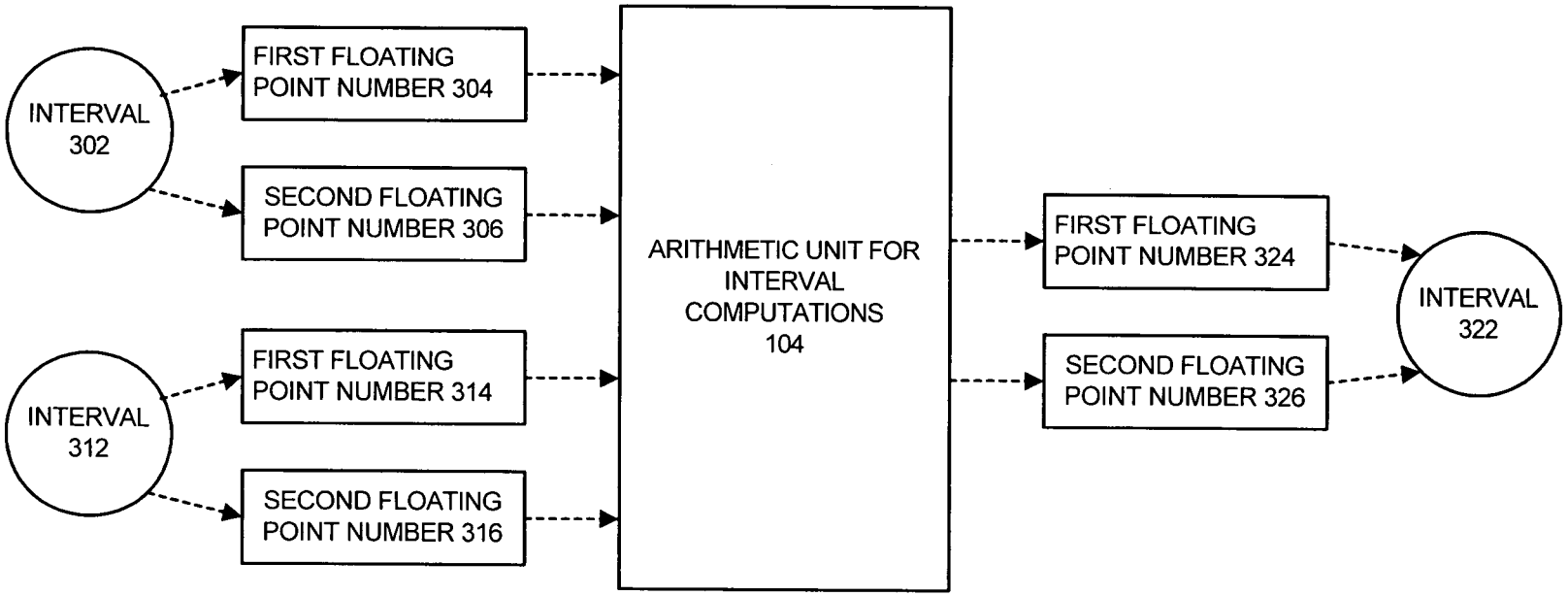
INTERVAL
302
FIRST FLOATING
POINT NUMBER 304
SECOND FLOATING
POINT NUMBER 306
INTERVAL
312
FIRST FLOATING
POINT NUMBER 314
SECOND FLOATING
POINT NUMBER 316
ARITHMETIC UNIT FOR
INTERVAL
COMPUTATIONS
104
FIRST FLOATING
POINT NUMBER 324
SECOND FLOATING
POINT NUMBER 326
INTERVAL
322

FIG. 3

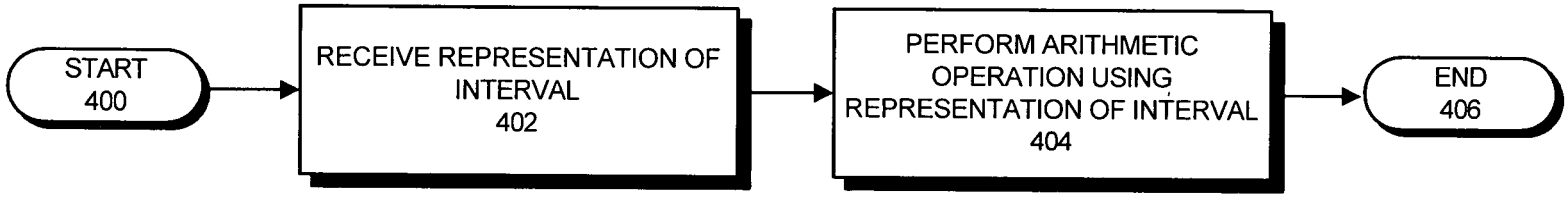
START
400
RECEIVE REPRESENTATION OF
INTERVAL
402
PERFORM ARITHMETIC
OPERATION USING
REPRESENTATION OF INTERVAL
404
END
406

FIG. 4

$$X \equiv \left[\underline{x}, \bar{x}\right] \equiv \left\{x \in \Re^* | \underline{x} \le x \le \bar{x}\right\}$$

$$Y \equiv \left[\underline{y}, \bar{y}\right] \equiv \left\{y \in \Re^* | \underline{y} \le y \le \bar{y}\right\}$$

(1) $$X + Y = \left[\downarrow \underline{x} + \underline{y}, \uparrow \bar{x} + \bar{y}\right]$$

(2) $$X - Y = \left[\downarrow \underline{x} - \bar{y}, \uparrow \bar{x} - \underline{y}\right]$$

(3) $$X \times Y = \left[\min\left(\downarrow \underline{x} \times \underline{y}, \underline{x} \times \bar{y}, \bar{x} \times \underline{y}, \bar{x} \times \bar{y}\right), \max\left(\uparrow \underline{x} \times \underline{y}, \underline{x} \times \bar{y}, \bar{x} \times \underline{y}, \bar{x} \times \bar{y}\right)\right]$$

(4) $$X / Y = \left[\min\left(\downarrow \underline{x} / \underline{y}, \underline{x} / \bar{y}, \bar{x} / \underline{y}, \bar{x} / \bar{y}\right), \max\left(\uparrow \underline{x} / \underline{y}, \underline{x} / \bar{y}, \bar{x} / \underline{y}, \bar{x} / \bar{y}\right)\right], \text{ if } 0 \notin Y$$

$$X / Y \subseteq \Re^*, \text{ if } 0 \in Y$$

FIG. 5

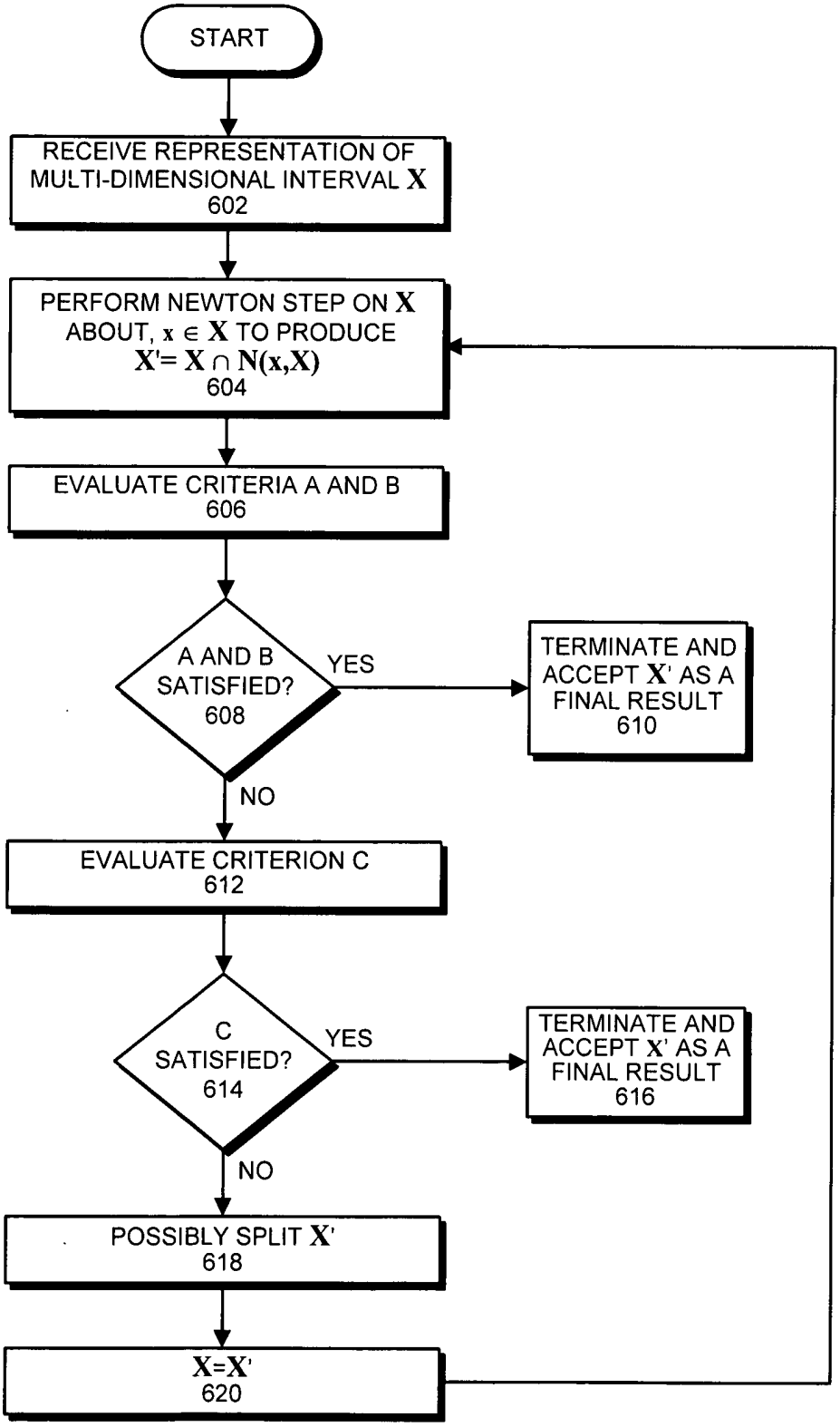
START
RECEIVE REPRESENTATION OF MULTI-DIMENSIONAL INTERVAL X
602
PERFORM NEWTON STEP ON X ABOUT, x ∈ X TO PRODUCE X'= X ∩ N(x,X)
604
EVALUATE CRITERIA A AND B
606
A AND B SATISFIED?
608
YES
TERMINATE AND ACCEPT X' AS A FINAL RESULT
610
NO
EVALUATE CRITERION C
612
C SATISFIED?
614
YES
TERMINATE AND ACCEPT X' AS A FINAL RESULT
616
NO
POSSIBLY SPLIT X'
618
X=X'
620

FIG. 6

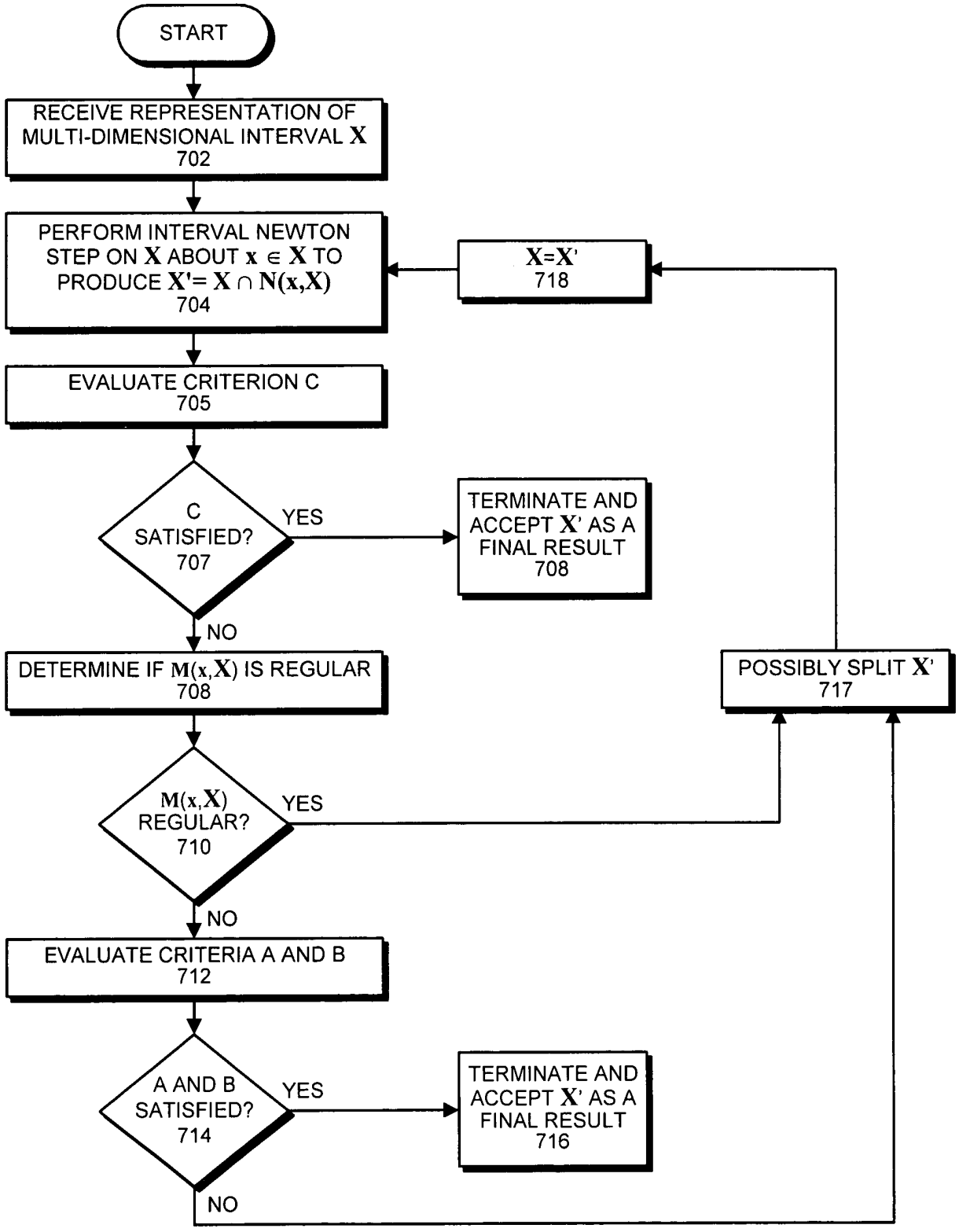
START
RECEIVE REPRESENTATION OF MULTI-DIMENSIONAL INTERVAL X
702
PERFORM INTERVAL NEWTON STEP ON X ABOUT x ∈ X TO PRODUCE X'= X ∩ N(x,X)
704
X=X'
718
EVALUATE CRITERION C
705
C SATISFIED?
707
YES
TERMINATE AND ACCEPT X' AS A FINAL RESULT
708
NO
DETERMINE IF M(x,X) IS REGULAR
708
POSSIBLY SPLIT X'
717
M(x,X) REGULAR?
710
YES
NO
EVALUATE CRITERIA A AND B
712
A AND B SATISFIED?
714
YES
TERMINATE AND ACCEPT X' AS A FINAL RESULT
716
NO

FIG. 7

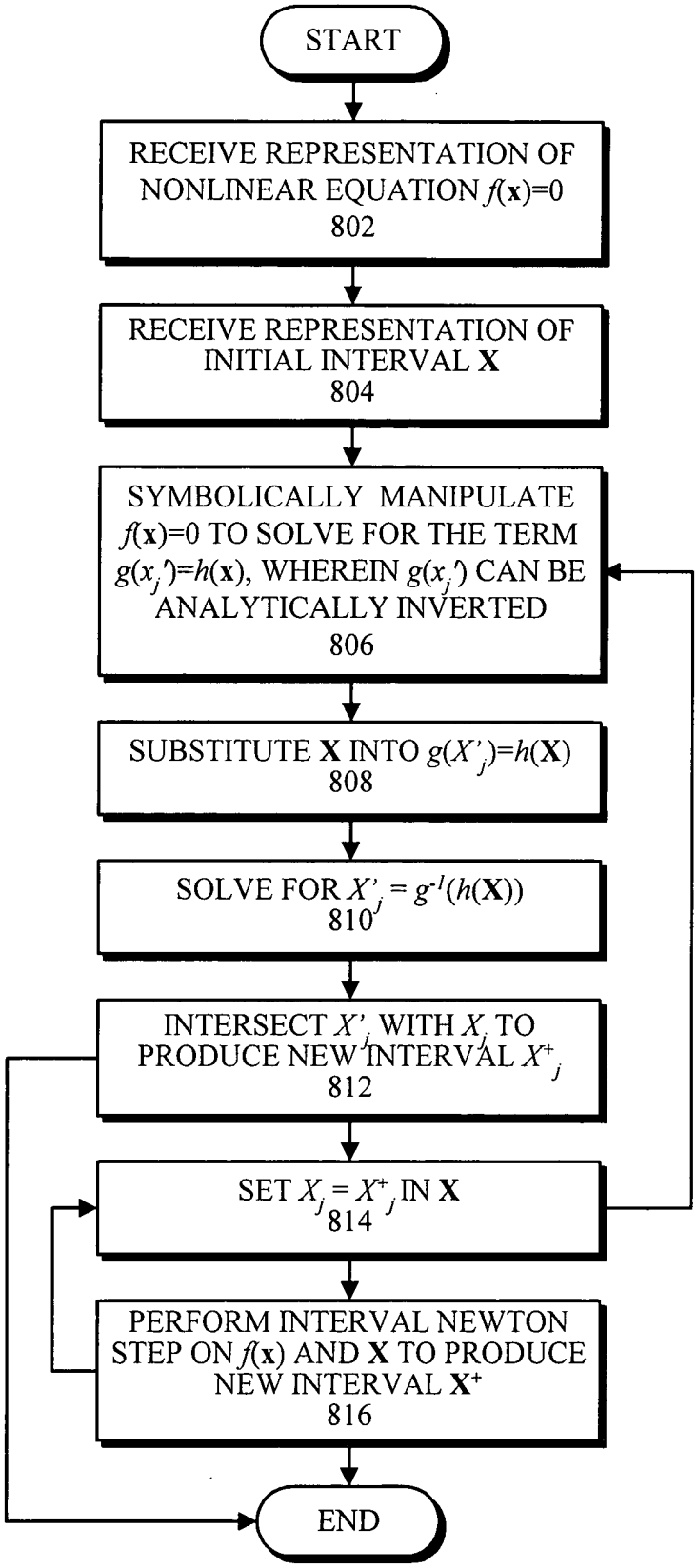
START
RECEIVE REPRESENTATION OF NONLINEAR EQUATION $f(\mathbf{x})=0$
802
RECEIVE REPRESENTATION OF INITIAL INTERVAL $\mathbf{X}$
804
SYMBOLICALLY MANIPULATE $f(\mathbf{x})=0$ TO SOLVE FOR THE TERM $g(x_j')=h(\mathbf{x})$, WHEREIN $g(x_j')$ CAN BE ANALYTICALLY INVERTED
806
SUBSTITUTE $\mathbf{X}$ INTO $g(X_j')=h(\mathbf{X})$
808
SOLVE FOR $X_j' = g^{-1}(h(\mathbf{X}))$
810
INTERSECT X_j' WITH X_j TO PRODUCE NEW INTERVAL X_j^+
812
SET $X_j = X_j^+$ IN $\mathbf{X}$
814
PERFORM INTERVAL NEWTON STEP ON $f(\mathbf{x})$ AND $\mathbf{X}$ TO PRODUCE NEW INTERVAL $\mathbf{X}^+$
816
END

FIG. 8

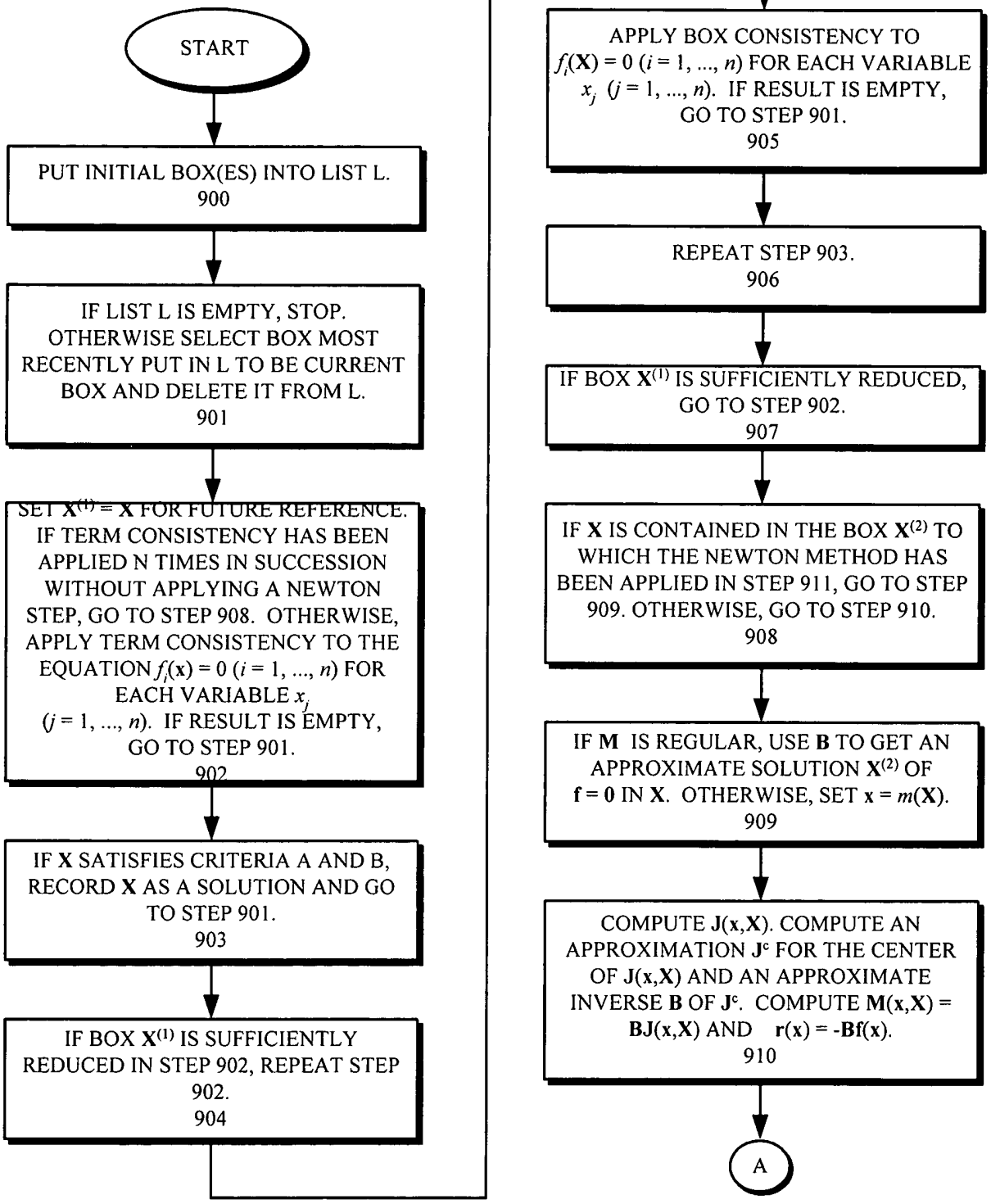
START
PUT INITIAL BOX(ES) INTO LIST L.
900
IF LIST L IS EMPTY, STOP. OTHERWISE SELECT BOX MOST RECENTLY PUT IN L TO BE CURRENT BOX AND DELETE IT FROM L.
901
SET $\mathbf{X}^{(1)} = \mathbf{X}$ FOR FUTURE REFERENCE. IF TERM CONSISTENCY HAS BEEN APPLIED N TIMES IN SUCCESSION WITHOUT APPLYING A NEWTON STEP, GO TO STEP 908. OTHERWISE, APPLY TERM CONSISTENCY TO THE EQUATION $f_i(\mathbf{x}) = 0$ ($i = 1, ..., n$) FOR EACH VARIABLE x_j ($j = 1, ..., n$). IF RESULT IS EMPTY, GO TO STEP 901.
902
IF **X** SATISFIES CRITERIA A AND B, RECORD **X** AS A SOLUTION AND GO TO STEP 901.
903
IF BOX $\mathbf{X}^{(1)}$ IS SUFFICIENTLY REDUCED IN STEP 902, REPEAT STEP 902.
904
APPLY BOX CONSISTENCY TO $f_i(\mathbf{X}) = 0$ ($i = 1, ..., n$) FOR EACH VARIABLE x_j ($j = 1, ..., n$). IF RESULT IS EMPTY, GO TO STEP 901.
905
REPEAT STEP 903.
906
IF BOX $\mathbf{X}^{(1)}$ IS SUFFICIENTLY REDUCED, GO TO STEP 902.
907
IF **X** IS CONTAINED IN THE BOX $\mathbf{X}^{(2)}$ TO WHICH THE NEWTON METHOD HAS BEEN APPLIED IN STEP 911, GO TO STEP 909. OTHERWISE, GO TO STEP 910.
908
IF **M** IS REGULAR, USE **B** TO GET AN APPROXIMATE SOLUTION $\mathbf{X}^{(2)}$ OF $\mathbf{f} = \mathbf{0}$ IN **X**. OTHERWISE, SET $\mathbf{x} = m(\mathbf{X})$.
909
COMPUTE **J**(**x**,**X**). COMPUTE AN APPROXIMATION $\mathbf{J}^c$ FOR THE CENTER OF **J**(**x**,**X**) AND AN APPROXIMATE INVERSE **B** OF $\mathbf{J}^c$. COMPUTE **M**(**x**,**X**) = **BJ**(**x**,**X**) AND **r**(**x**) = -**Bf**(**x**).
910
A

FIG. 9A

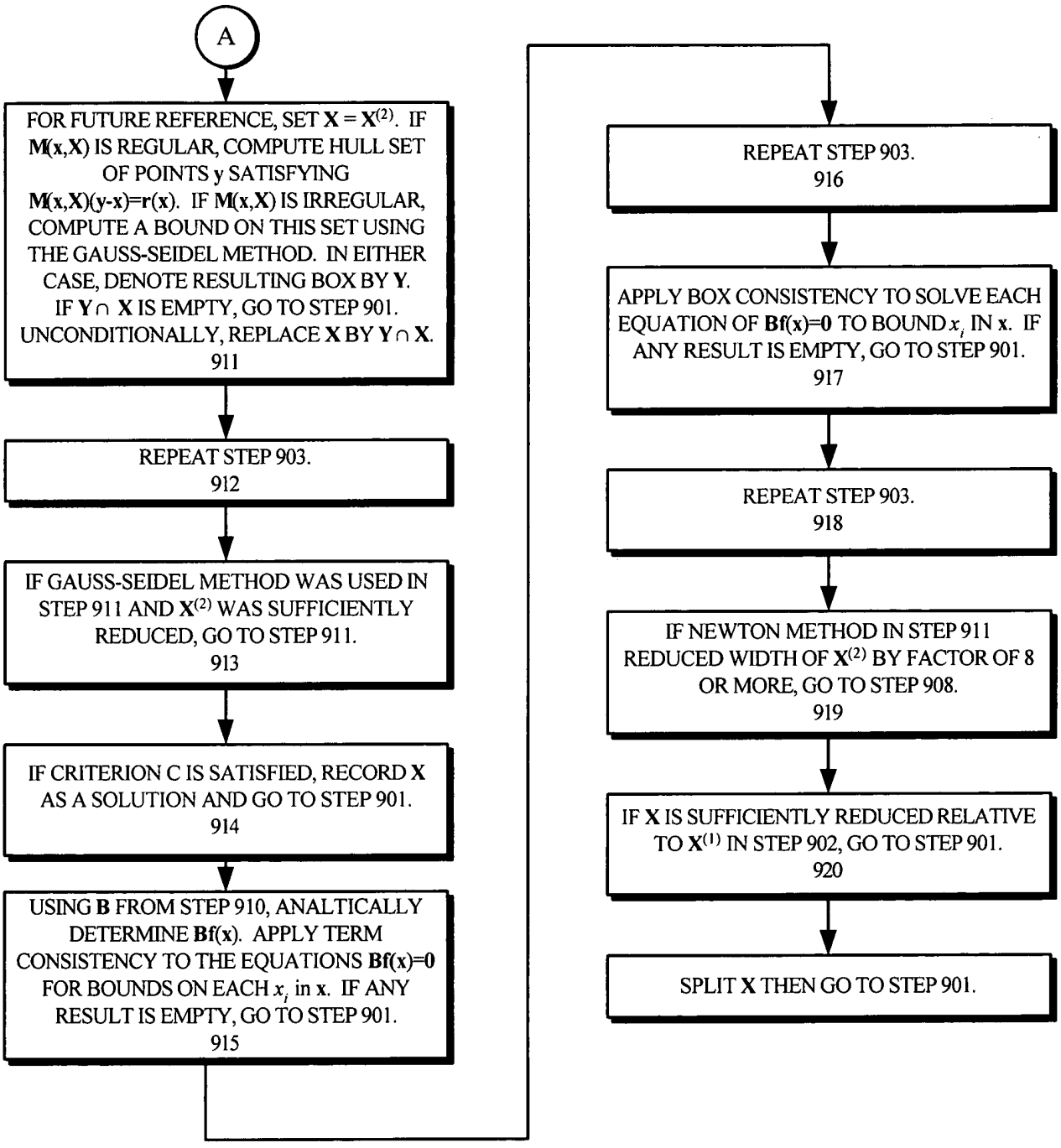
A
FOR FUTURE REFERENCE, SET X = X(2). IF M(x,X) IS REGULAR, COMPUTE HULL SET OF POINTS y SATISFYING M(x,X)(y-x)=r(x). IF M(x,X) IS IRREGULAR, COMPUTE A BOUND ON THIS SET USING THE GAUSS-SEIDEL METHOD. IN EITHER CASE, DENOTE RESULTING BOX BY Y. IF Y ∩ X IS EMPTY, GO TO STEP 901. UNCONDITIONALLY, REPLACE X BY Y ∩ X.
911
REPEAT STEP 903.
912
IF GAUSS-SEIDEL METHOD WAS USED IN STEP 911 AND X(2) WAS SUFFICIENTLY REDUCED, GO TO STEP 911.
913
IF CRITERION C IS SATISFIED, RECORD X AS A SOLUTION AND GO TO STEP 901.
914
USING B FROM STEP 910, ANALTICALLY DETERMINE Bf(x). APPLY TERM CONSISTENCY TO THE EQUATIONS Bf(x)=0 FOR BOUNDS ON EACH x_i in x. IF ANY RESULT IS EMPTY, GO TO STEP 901.
915
REPEAT STEP 903.
916
APPLY BOX CONSISTENCY TO SOLVE EACH EQUATION OF Bf(x)=0 TO BOUND x_i IN x. IF ANY RESULT IS EMPTY, GO TO STEP 901.
917
REPEAT STEP 903.
918
IF NEWTON METHOD IN STEP 911 REDUCED WIDTH OF X(2) BY FACTOR OF 8 OR MORE, GO TO STEP 908.
919
IF X IS SUFFICIENTLY REDUCED RELATIVE TO X(1) IN STEP 902, GO TO STEP 901.
920
SPLIT X THEN GO TO STEP 901.

FIG. 9B